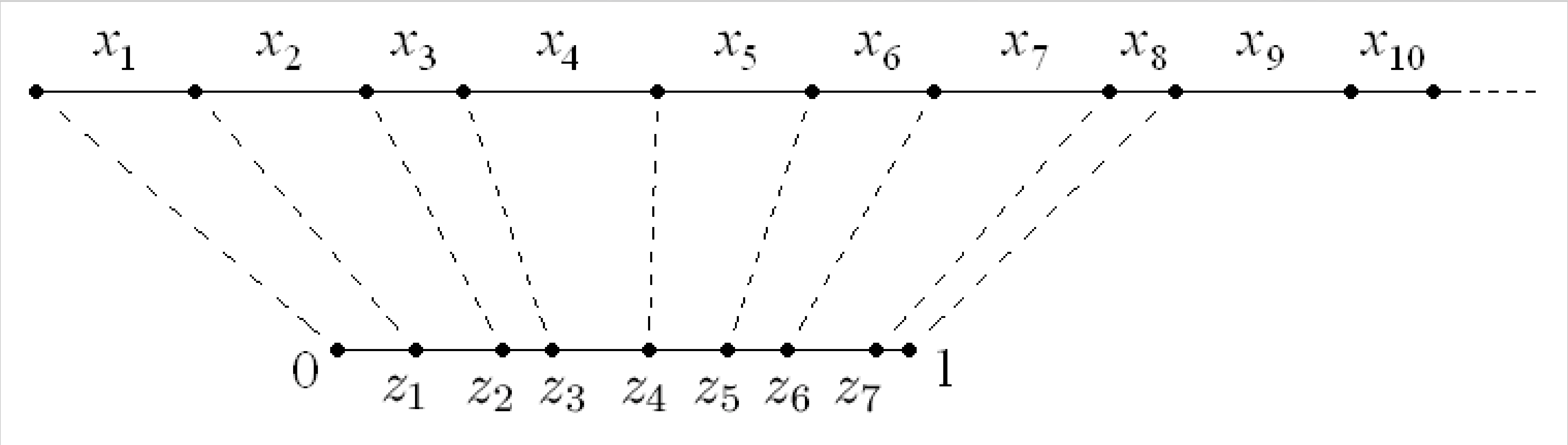
# Exact Analytical Calculation for the Percolation Crossover in Deterministic Partially Self-Avoiding Walks in One-Dimensional Random Media

arXiv:cond-mat/0702030v1 [February 1, 2007]




$$g(x) = \begin{cases} re^{-rx} & \text{for } x \geq 0 \\ 0 & \text{for } x < 0 \end{cases}$$

# Probability to Visit n Points

$$
\begin{aligned}
S^{(\infty)}_{\mu,si}(n) \quad = \quad & \prod_{j=1}^{\mu} \int_0^\infty \mathrm{d}x_j \; r e^{-r x_j} \\
& \prod_{j=\mu+1}^{n-1} \int_0^{\sum_{k=j-\mu}^{j-1} x_k} \mathrm{d}x_j \; r e^{-r x_j} \\
& \int_{\sum_{k=n-\mu}^{n-1} x_k}^{\infty} \mathrm{d}x_n \; r e^{-r x_n}
\end{aligned}
$$

# Recursive Calculation

$$y_j = e^{-r x_j}$$

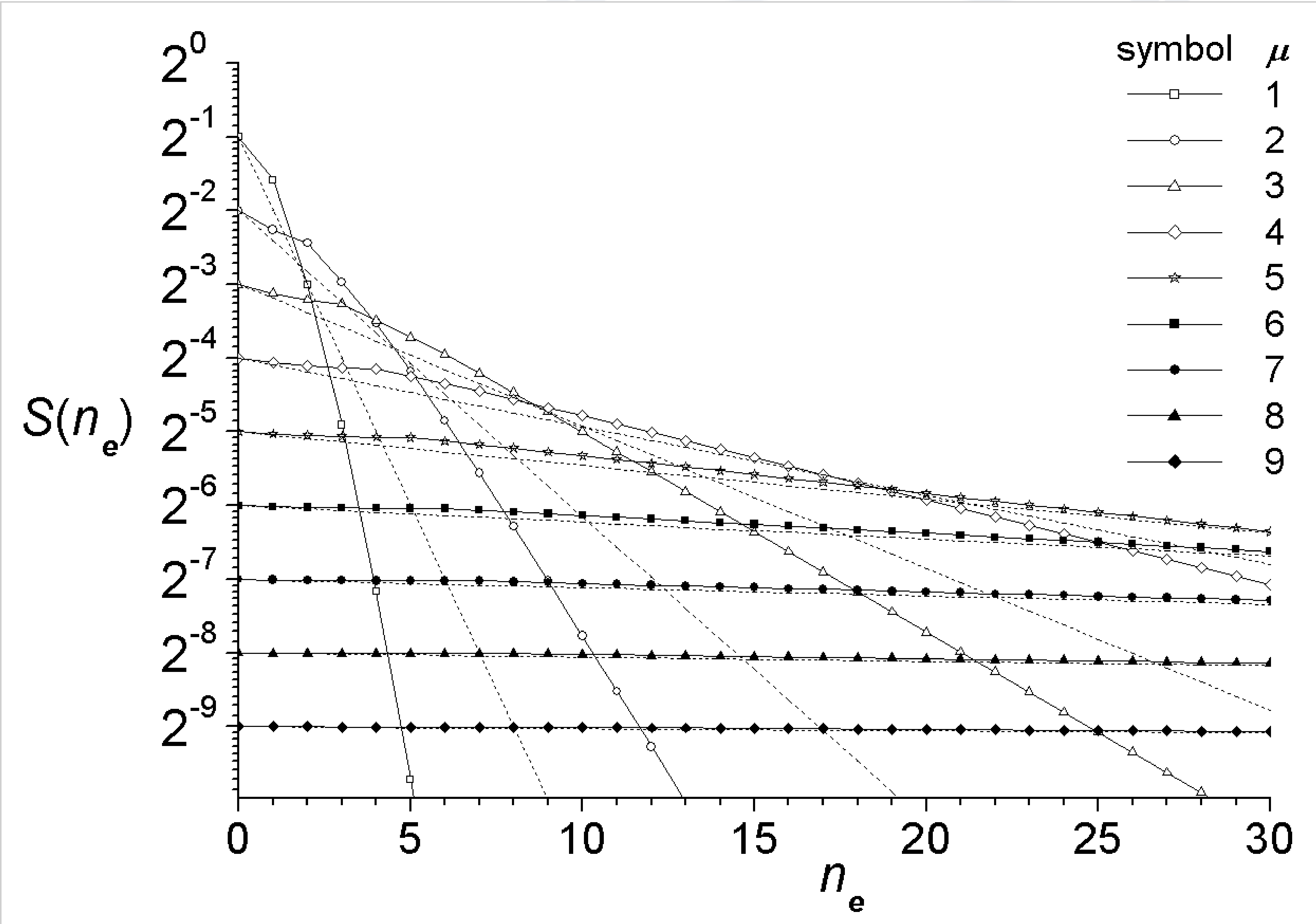
$$S_{\mu,si}^{(\infty)}(n) = \prod_{j=1}^{n} \mathcal{I}_j$$

$$\mathcal{I}_j = \begin{cases} \int_0^1 \mathrm{d}y_j & \text{for } 1 \le j \le \mu \\ \int_{\tilde{y}_j}^1 \mathrm{d}y_j & \text{for } \mu+1 \le j \le n-1 \\ \int_0^{\tilde{y}_j} \mathrm{d}y_j & \text{for } j = n \end{cases}$$

$y_4^1 y_5^1 y_6^1$

- $+\frac{y_3^0 y_4^1 y_5^1}{2}$
  - $+\frac{y_2^0 y_3^0 y_4^1}{2\cdot2}$
    - $+\frac{y_1^0 y_2^0 y_3^0}{2\cdot2\cdot2} \longrightarrow +\frac{1}{2\cdot2\cdot2\cdot1\cdot1\cdot1}$
    - $-\frac{y_1^2 y_2^2 y_3^2}{2\cdot2\cdot2} \longrightarrow -\frac{1}{2\cdot2\cdot2\cdot3\cdot3\cdot3}$
  - $-\frac{y_2^2 y_3^2 y_4^3}{2\cdot2}$
    - $-\frac{y_1^0 y_2^2 y_3^2}{2\cdot2\cdot4} \longrightarrow -\frac{1}{2\cdot2\cdot4\cdot1\cdot3\cdot3}$
    - $+\frac{y_1^4 y_2^6 y_3^6}{2\cdot2\cdot4} \longrightarrow +\frac{1}{2\cdot2\cdot4\cdot5\cdot7\cdot7}$
- $-\frac{y_3^2 y_4^3 y_5^3}{2}$
  - $-\frac{y_2^0 y_3^2 y_4^3}{2\cdot4}$
    - $-\frac{y_1^0 y_2^0 y_3^2}{2\cdot4\cdot4} \longrightarrow -\frac{1}{2\cdot4\cdot4\cdot1\cdot1\cdot3}$
    - $+\frac{y_1^4 y_2^4 y_3^6}{2\cdot4\cdot4} \longrightarrow +\frac{1}{2\cdot4\cdot4\cdot5\cdot5\cdot7}$
  - $+\frac{y_2^4 y_3^6 y_4^7}{2\cdot4}$
    - $+\frac{y_1^0 y_2^4 y_3^6}{2\cdot4\cdot8} \longrightarrow +\frac{1}{2\cdot4\cdot8\cdot1\cdot5\cdot7}$
    - $-\frac{y_1^8 y_2^{12} y_3^{14}}{2\cdot4\cdot8} \longrightarrow -\frac{1}{2\cdot4\cdot8\cdot9\cdot13\cdot15}$

$\mathcal{I}_6$ $\quad$ $\mathcal{I}_5$ $\quad$ $\mathcal{I}_4$ $\quad$ $\mathcal{I}_3, \mathcal{I}_2, \mathcal{I}_1$


symbol μ
1
2
3
4
5
6
7
8
9
$S(n_e)$
$2^0$
$2^{-1}$
$2^{-2}$
$2^{-3}$
$2^{-4}$
$2^{-5}$
$2^{-6}$
$2^{-7}$
$2^{-8}$
$2^{-9}$
0
5
10
15
20
25
30
$n_e$

$n_e = n - (\mu + 1)$

# Mean Field Approach

$$\tilde{y} = \prod_{k=1}^{\mu} y_k$$

$$\tilde{w} = -\ln \tilde{y} = \sum_{k=1}^{\mu} w_k$$

$$p(\tilde{w}) = \frac{\tilde{w}^{\mu-1} e^{-\tilde{w}}}{\Gamma(\mu)}$$

$$|p(\tilde{y}) \mathrm{d}\tilde{y}| = |p(\tilde{w}) \mathrm{d}\tilde{w}|$$

$$p(\tilde{y}) = \frac{(-\ln \tilde{y})^{\mu-1}}{\Gamma(\mu)}$$

$$\langle \tilde{y}^m \rangle = (m+1)^{-\mu}$$

# Mean Field Solution

$$\mathcal{I}_j \approx \begin{cases} \int_0^1 \mathrm{d}y_j \ , & \text{for } 0 \leq j \leq \mu \\ \int_{2^{-\mu}}^1 \mathrm{d}y_j \ , & \text{for } \mu + 1 \leq j \leq n-1 \\ \int_0^{2^{-\mu}} \mathrm{d}y_j \ , & \text{for } j = n \end{cases}$$

$$S_{\mu,si}^{(\infty)}(n) \approx 2^{-\mu}(1 - 2^{-\mu})^{n-\mu-1}$$

$$\langle n \rangle = 2^{\mu} + \mu$$

$$\Delta n = \sqrt{2^{2\mu} - 2^{\mu}}$$

# Nomenclature

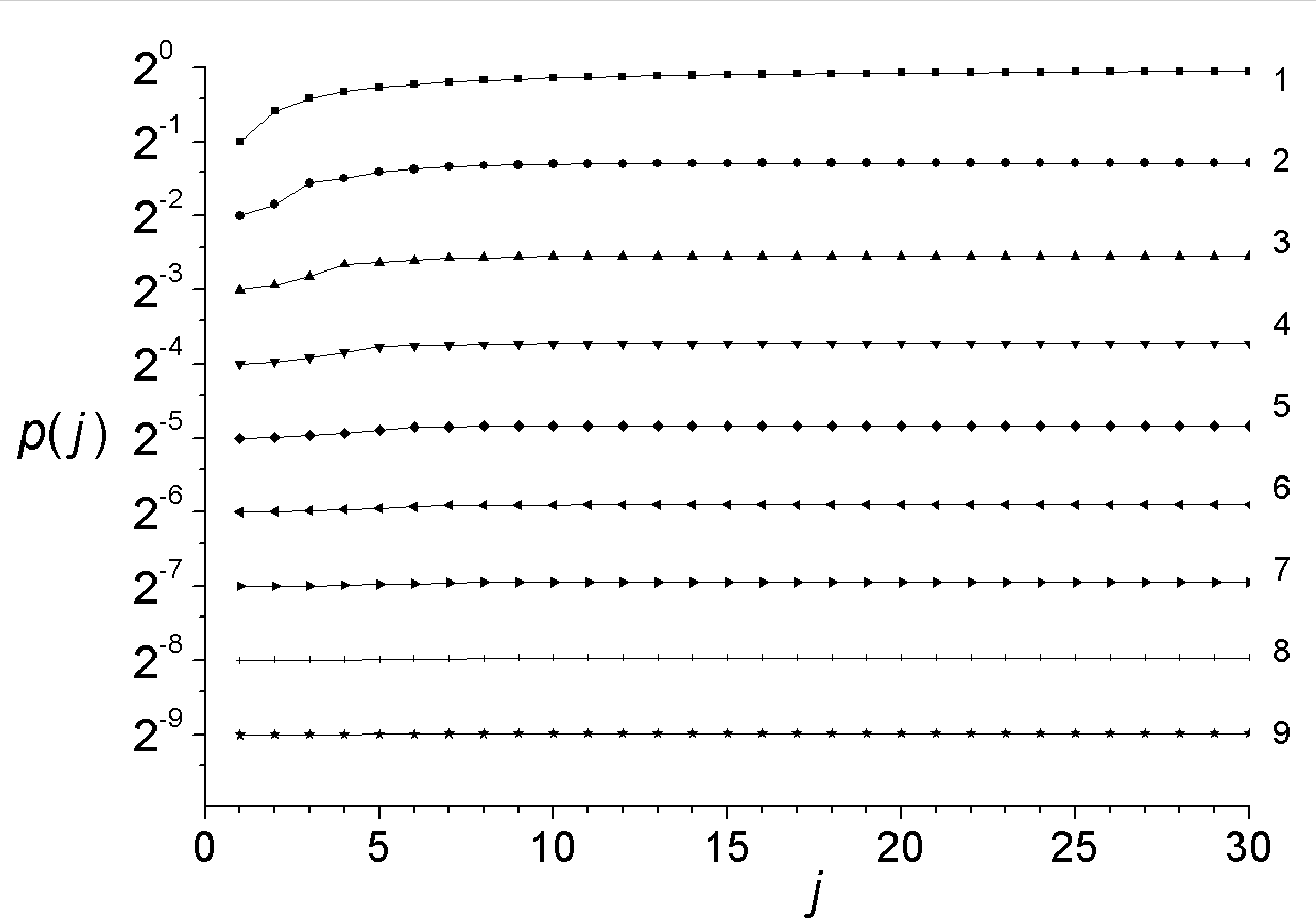
$$\overline{F}^{(\infty)}_{\mu,si}(n) = \sum_{k=n}^{\infty} S^{(\infty)}_{\mu,si}(k)$$

$$p_\mu(j) = \frac{S^{(\infty)}_{\mu,si}(n=\mu+j)}{\overline{F}^{(\infty)}_{\mu,si}(n=\mu+j)} \approx \frac{1}{2^\mu}$$

$$\overline{F}^{(\infty)}_{\mu,si}(n) \approx (1-2^{-\mu})^{n-\mu-1}$$

$$q_\mu(j) \approx 1-2^{-\mu}$$

## Return Probabilities for Varying μ



# Analytical Solution for μ=1

$$S_{1,si}^{(\infty)}(n_e) = \frac{n_e+1}{(n_e+2)!}$$

$$\overline{F}_{1,si}^{(\infty)}(n_e) = \frac{1}{(n_e+1)!}$$

$$p_1(j) = \frac{j}{j+1}$$

# Percolation Probability

$$P_N(\mu) = q_\mu^{N-(\mu+1)} = (1 - 2^{-\mu})^{N-\mu-1}$$

$$\begin{aligned} \mathrm{d}_\mu^2 P_N(\mu)|_{\mu_1^c} &= 0 \\ \mu_1^c &= \log_2 N \\ \varepsilon &= \tfrac{e}{\ln 2} \approx 3.92 \end{aligned}$$

# Percolation Probabilities for Fixed N