Semi-Infinite Disordered Media

\[ g(x) = \begin{cases} 
  re^{-rx} & \text{for } x \geq 0 \\
  0 & \text{for } x < 0 
\end{cases} \]
Probability to Visit n Points

\[ S_{\mu,si}^{(\infty)}(n) = \prod_{j=1}^{\mu} \int_{0}^{\infty} \frac{d x_j}{x_j} e^{-r x_j} \prod_{j=\mu+1}^{n-1} \int_{0}^{\infty} \frac{d x_j}{x_j} e^{-r x_j} \int_{\infty}^{\sum_{k=n-\mu}^{n-1} x_k} d x_n e^{-r x_n} \]
Recursive Calculation

\[ y_j = e^{-rx_j} \]

\[ S^{(\infty)}_{\mu, s_1}(n) = \prod_{j=1}^{n} I_j \]

\[ I_j = \begin{cases} 
\int_{0}^{1} dy_j & \text{for } 1 \leq j \leq \mu \\
\int_{y_j}^{1} dy_j & \text{for } \mu + 1 \leq j \leq n - 1 \\
\int_{0}^{\tilde{y}_j} dy_j & \text{for } j = n 
\end{cases} \]

\[ I_6 = 1 \]
\[ I_5 = \frac{1}{2} \]
\[ I_4 = \frac{1}{4} \]
\[ I_3, I_2, I_1 = \frac{1}{8} \]

\[ S^{(\infty)}_{\mu, s_1}(n) = I_1 \times I_2 \times I_3 \times I_4 \times I_5 \times I_6 \]

\[ y_j = e^{-r x_j} \]

\[ S^{(\infty)}_{\mu, s_1}(n) = \prod_{j=1}^{n} I_j \]

\[ y_j = e^{-r x_j} \]
Probability Distribution of $n_e$ for Varying $\mu$

\[ S(n_e) = n - (\mu + 1) \]
Mean Field Approach

\[ \tilde{y} = \prod_{k=1}^{\mu} y_k \]
\[ \tilde{w} = -\ln \tilde{y} = \sum_{k=1}^{\mu} w_k \]
\[ p(\tilde{w}) = \frac{\tilde{w}^{\mu-1} e^{-\tilde{w}}}{\Gamma(\mu)} \]
\[ |p(\tilde{y})d\tilde{y}| = |p(\tilde{w})d\tilde{w}| \]
\[ p(\tilde{y}) = \frac{(\ln \tilde{y})^{\mu-1}}{\Gamma(\mu)} \]
\[ \left\langle \tilde{y}^m \right\rangle = (m + 1)^{-\mu} \]
Mean Field Solution

\[ \langle n \rangle = 2^\mu + \mu \]

\[ \int_0^1 dy_j, \quad \text{for } 0 \leq j \leq \mu \]

\[ \int_{2^{-\mu}}^1 dy_j, \quad \text{for } \mu + 1 \leq j \leq n - 1 \]

\[ \int_{2^{-\mu}}^2 dy_j, \quad \text{for } j = n \]

\[ S_{\mu,si}^{(\infty)}(n) \approx 2^{-\mu}(1 - 2^{-\mu})^{n-\mu-1} \]

\[ \Delta n = \sqrt{2^{2\mu} - 2^\mu} \]
\[ F_{\mu, si}^{(\infty)}(n) = \sum_{k=n}^{\infty} S_{\mu, si}^{(\infty)}(k) \]

\[ F_{\mu, si}^{(\infty)}(n) \approx (1 - 2^{-\mu})^{n-\mu-1} \]

\[ p_\mu(j) = \frac{S_{\mu, si}^{(\infty)}(n=\mu+j)}{F_{\mu, si}^{(\infty)}(n=\mu+j)} \approx \frac{1}{2^\mu} \]

\[ q_\mu(j) \approx 1 - 2^{-\mu} \]
Return Probabilities for Varying $\mu$

where each functional $I_j$ is given by Eq. 3. The root node of Fig. 2 is now set to 1 (or, equivalently, $y_0 = y_0$ ... the return probability is $p_\mu = 1 - q_\mu = 2^{-\mu}$ and one has: $S(\infty)_{\mu,si} = 2^{-\mu}(1 - 2^{-\mu})^n$, which is the result of Eq. 8.
Analytical Solution for $\mu=1$

\[
S_{1,si}^{(\infty)}(n_e) = \frac{n_e+1}{(n_e+2)!}
\]

\[
F_{1,si}^{(\infty)}(n_e) = \frac{1}{(n_e+1)!}
\]

\[
p_1(j) = \frac{j}{j+1}
\]
Percolation Probability

\[ P_N(\mu) = q_\mu^{N-(\mu+1)} = (1 - 2^{-\mu})^{N-\mu-1} \]

\[ d_\mu^2 P_N(\mu)|_{\mu^c_1} = 0 \]

\[ \mu^c_1 = \log_2 N \]

\[ \varepsilon = \frac{e}{\ln 2} \approx 3.92 \]
Percolation Probabilities for Fixed N

The finite disordered medium is constructed by N points... Also, they can be applied to an infinite line segment, where...