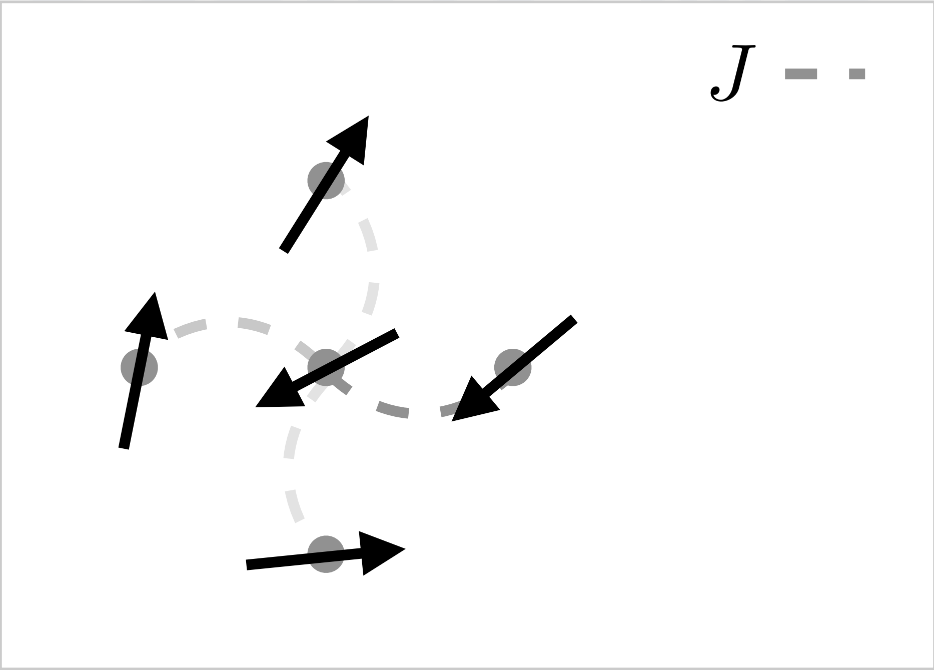
Finite Lattice Tiles and Scaling

Results with DCA and Lanczos

The Spin ½ XY Model

Studied via. Exact Diagonalization using the Lanczos Algorithm

$$H = 2J \sum_{\langle i,j \rangle} (S_i^x S_j^x + S_i^y S_j^y)$$



The Spin ½ Heisenberg Model

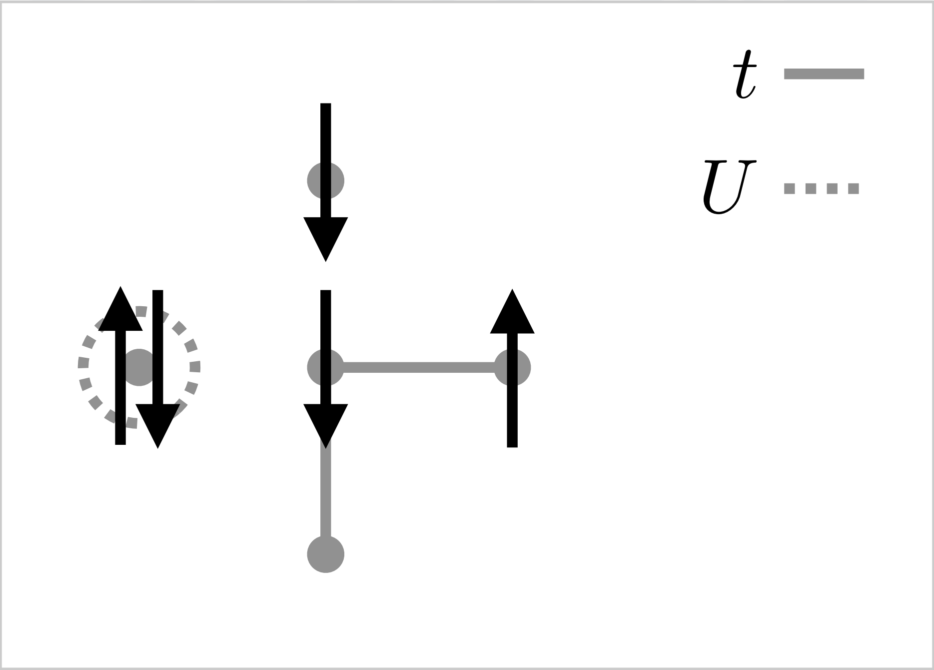
Studied via. Exact Diagonalization using the Lanczos Algorithm

$$H = 2J \sum_{\langle i,j \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

The Hubbard Model

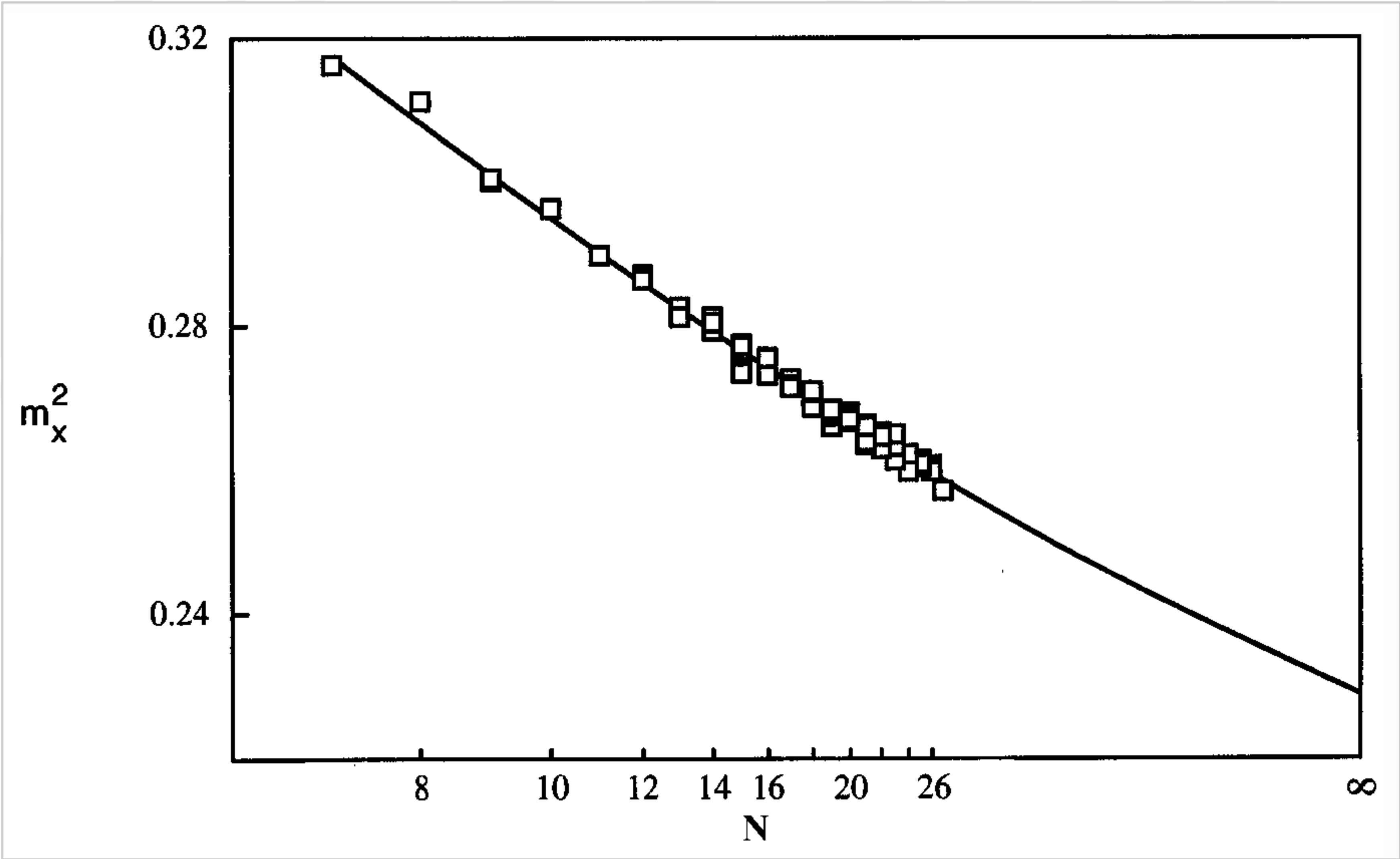
Studied via. Dynamic Cluster Approximation using the Hirsh-Fye Impurity Algorithm

$$H = -t \sum_{\langle i,j \rangle, \sigma} c^{\dagger}_{i\sigma} c_{j\sigma} + U \sum_i (n_{i\uparrow} - \frac{1}{2})(n_{i\downarrow} - \frac{1}{2})$$

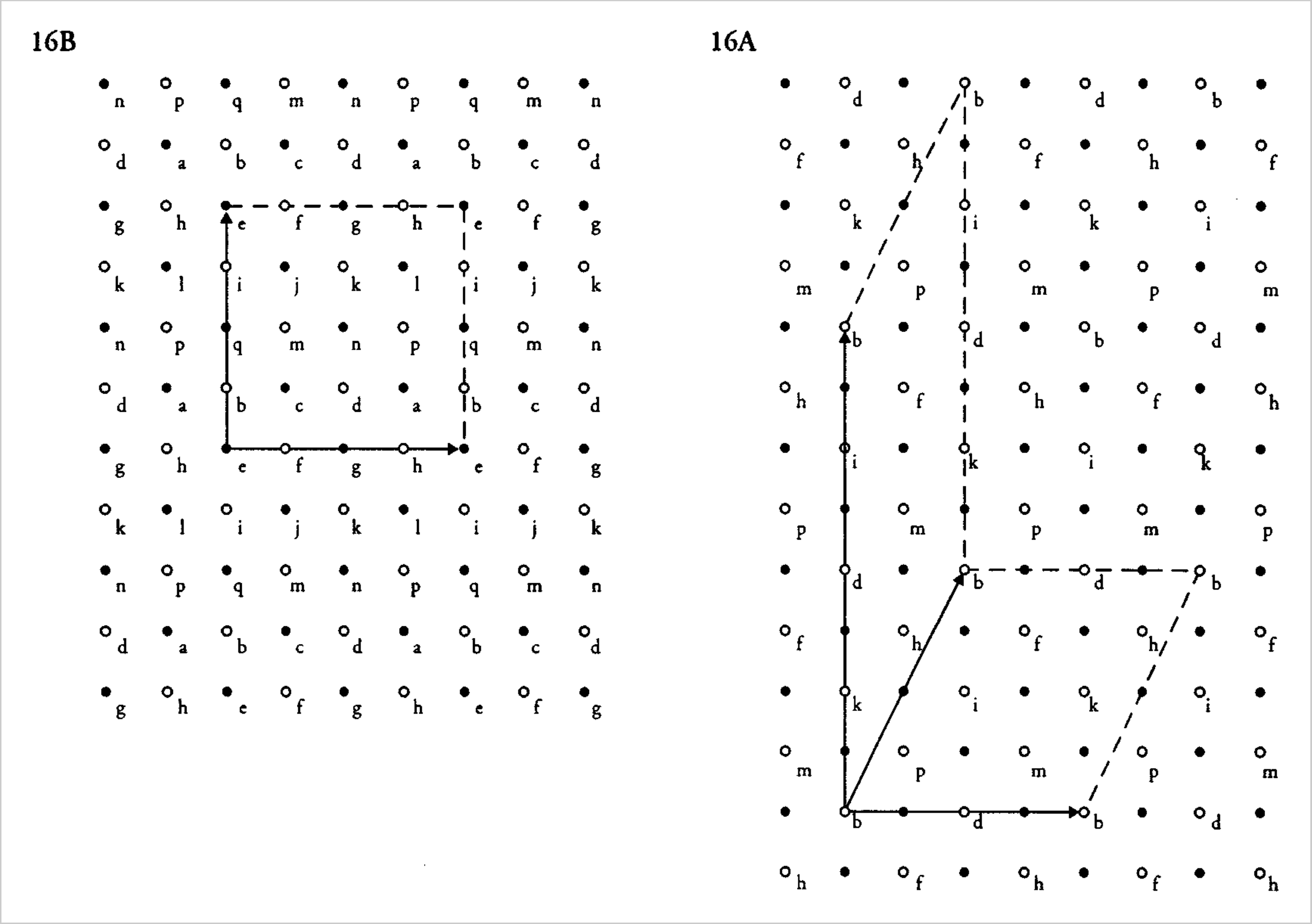


Finite Scaling Concerns

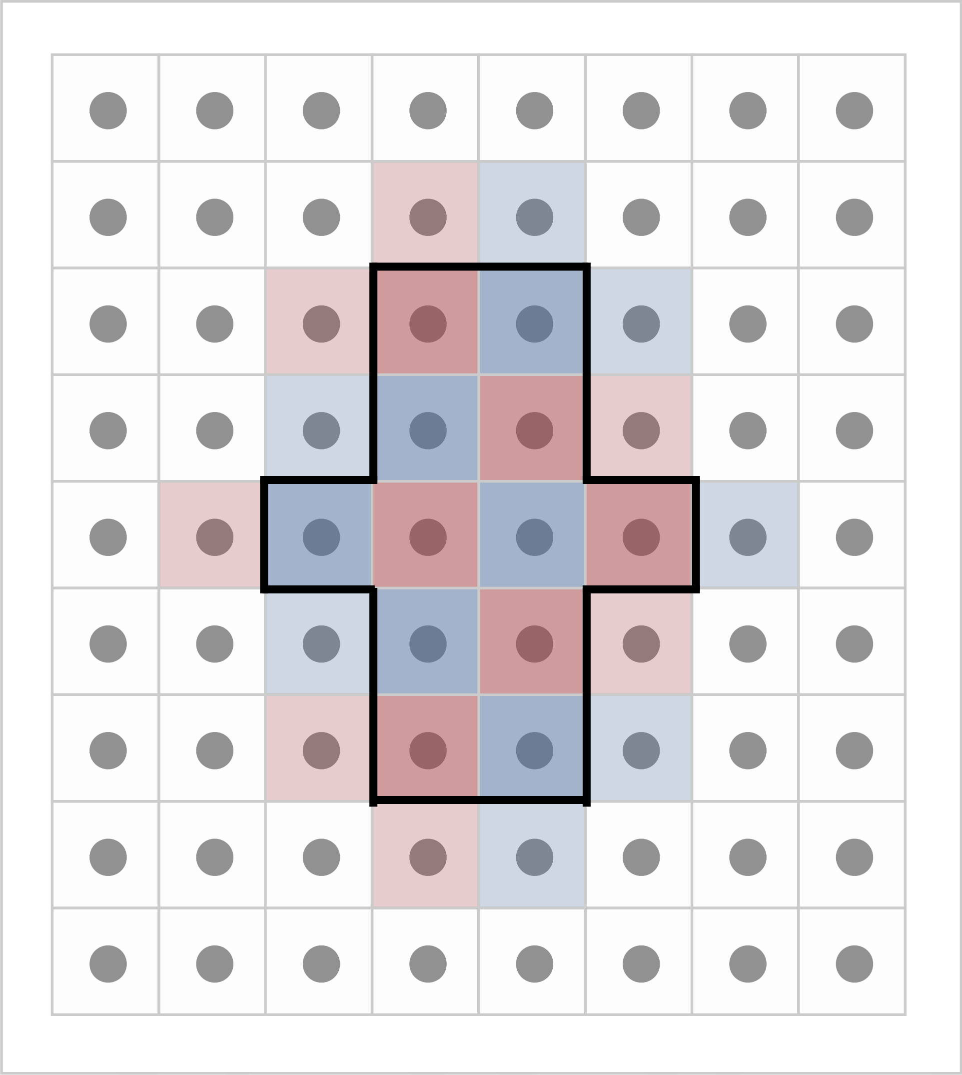
Motivation for Grading Tiles

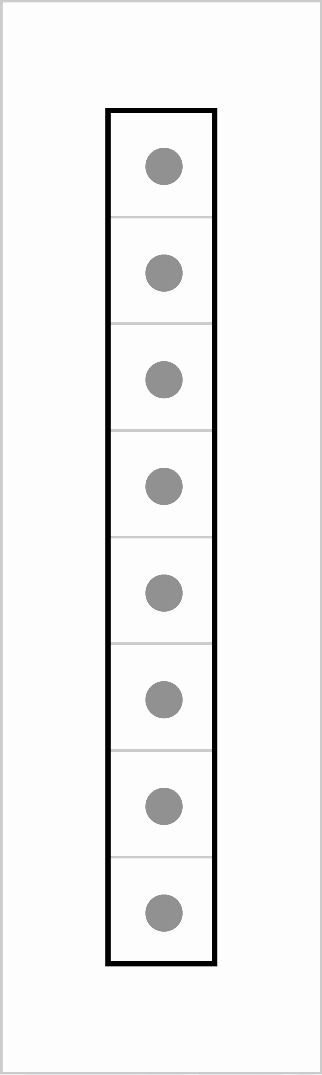


Finite Lattice Generation



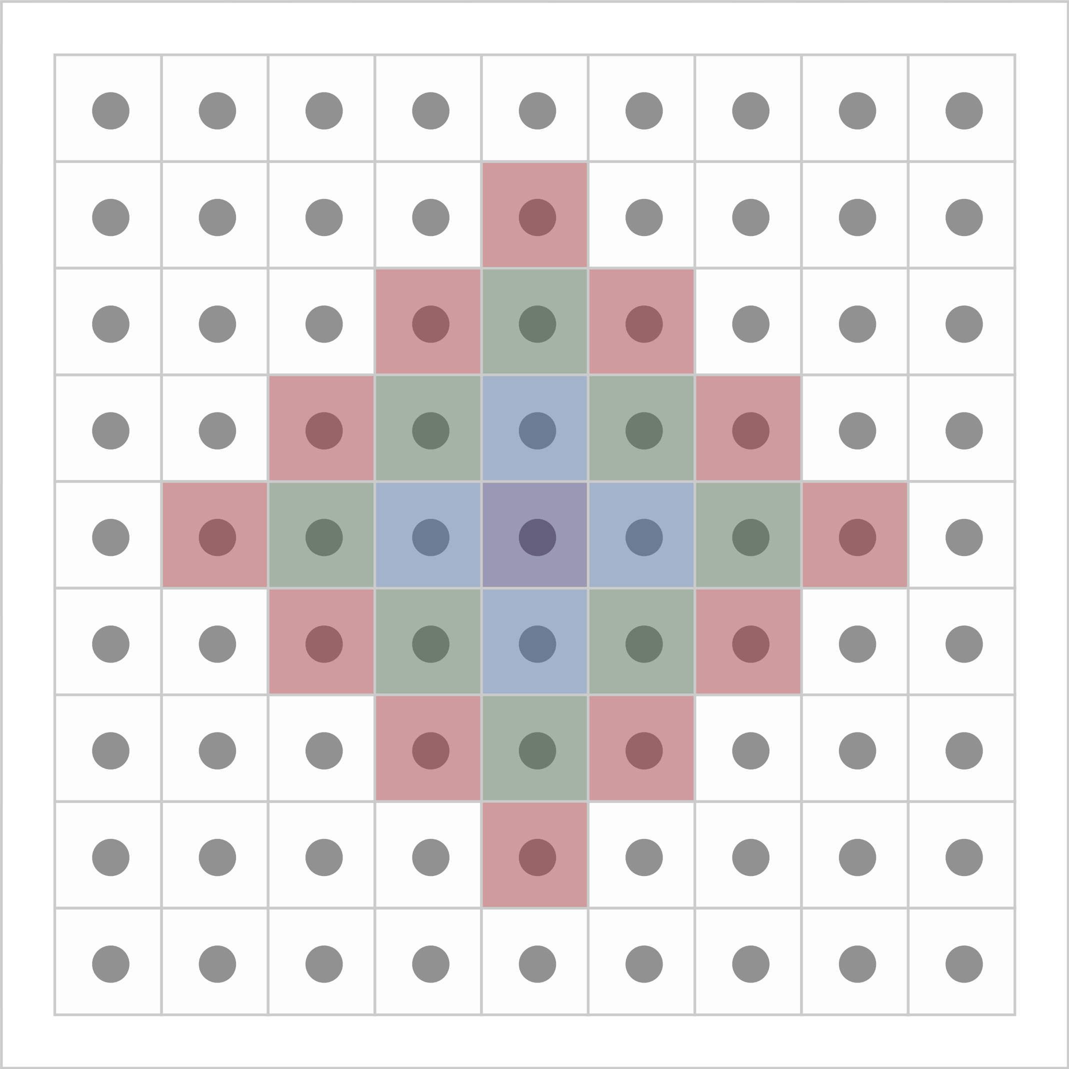
Poor Tiles





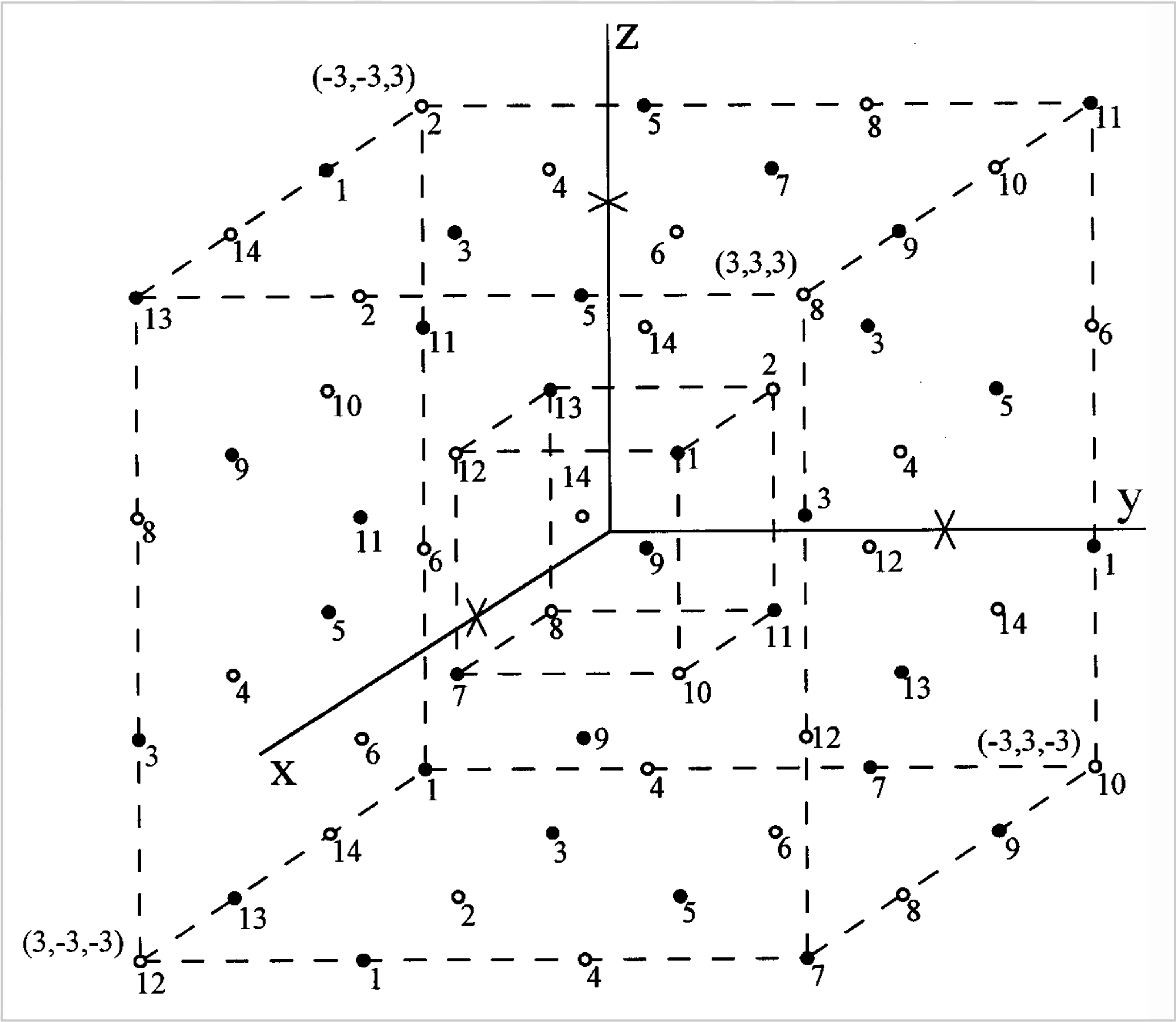
Grading Criteria for Tiles

Imperfection



Grading Criteria for Tiles

Symmetry



Grading Criteria for Tiles

$$l = (l_1 l_2 l_3)^{1/3}$$

$$d = (d_1 d_2 d_3 d_4)^{1/4}$$

$$f = (f_1 f_2 f_3 f_4 f_5 f_6)^{1/6}$$

$$c_1 = \frac{3^{1/2} l}{d}$$

$$c_2 = \frac{2^{1/2} l}{f}$$

$$C = max(c_1, c_1^{-1}) max(c_2, c_2^{-1})$$

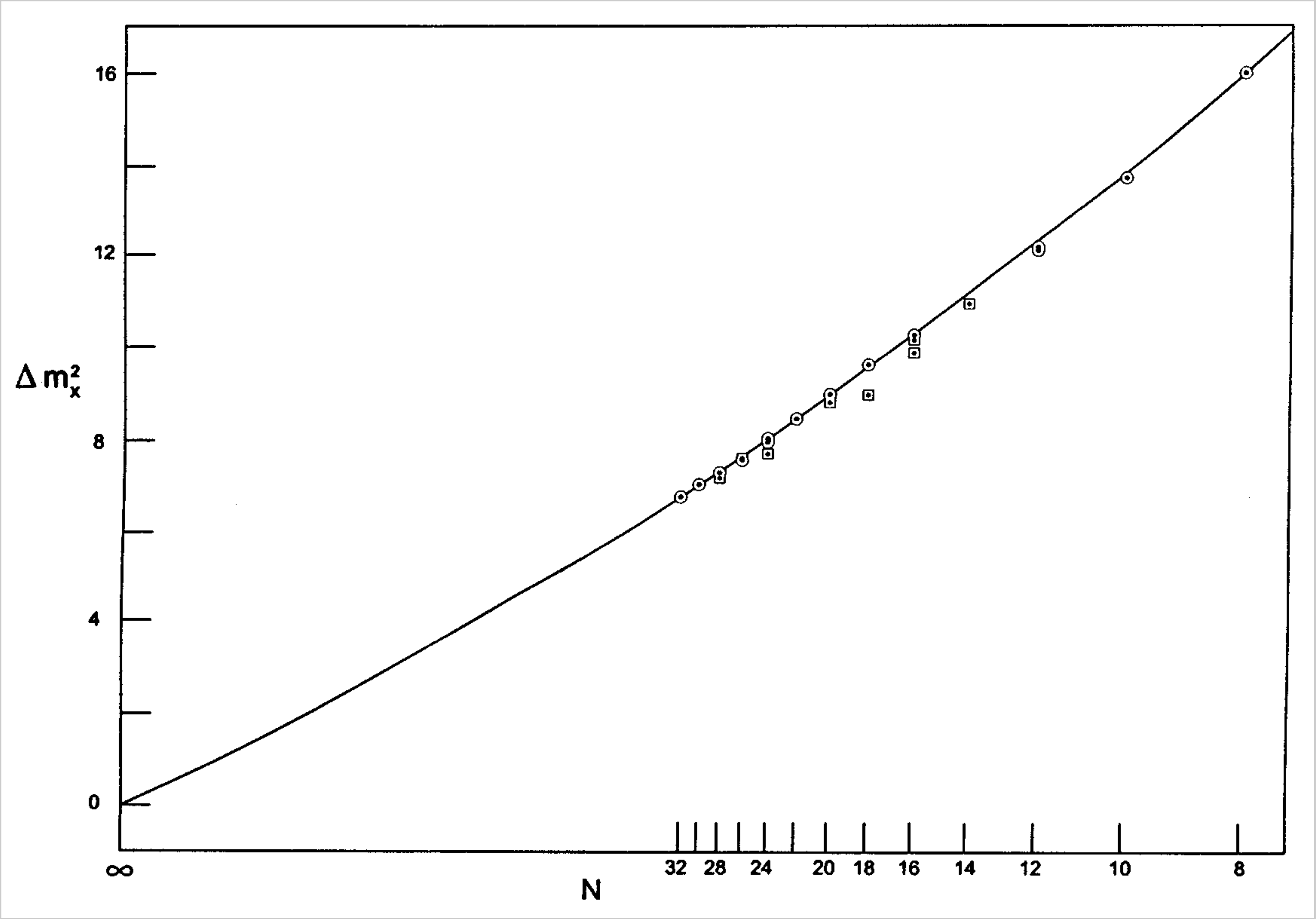
Lattice Grades

Lattice	$\mathscr{S}$	$\mathscr{I}$	Property that deviates ε_o^{XY}	m_x^{XY}	ε_o^{HA}
8A	C_{2h}	1	A	B	
10A	C_i	0	A^-	B^+	
12A	C_{2h}	0	A^+	B^+	
12B	C_{2h}	0	B^+	A^-	
14A	C_{3i}	0	B^+	B	A^-
14B	C_i	0	B^+	A	
14C	C_i	0	A^-	B^+	
16A*	O_h	0	B^+	B^+	A^-
16B*	D_{2h}	0	B^+	B^+	A^-
16C*	D_{2h}	0	B^+	B^+	A^-
16D	C_{2h}	0	B^+	B^+	A^-
16E	C_{2h}	0	A^+	B^+	
16F	C_{2h}	1	B	B^+	
18A*	D_{3h}	0	A^-	A^-	A^-
18B*	C_{3i}	0	A^-	A^-	A^-
18C	C_i	0	B^+	B^+	A^-
18D	C_{2h}	0	B^+	B^+	
18E	C_{2h}	0	A	B^+	
18F	C_{2h}	2	B	B^+	
20A	C_i	0	A^-	A^-	A^-
20B*	C_i	0	A^-	B^+	A^-
20C*	C_i	0	A^-	B^+	A^-
20D*	C_i	0	A^-	B^+	A^-
20E	C_{2h}	0	A^+	A	B^+
20F	C_{2h}	0	A^-	A^+	
20G	C_{2h}	1	B^+	B^+	
22A	C_i	0	A^-	B^+	B^+
22B	C_i	2	A^-	B^+	B^+
22C	C_i	0	A	A^-	
22D	C_i	0	B^+	B^+	
22E	C_i	2	B^+	B^+	
24A	C_{2h}	2	A	A^-	A^+
24B	D_{3h}	2	A^-	A^-	A^+
24C	C_i	0	A^-	B^+	A^-
24D	D_{2h}	4	A^-	A	
24E	C_{2h}	0	B^+	B^+	
26A	C_i	2	A^+	A^-	A^-
26B	C_{3i}	0	A^-	B^+	A^-
26C	C_i	0	B^+	B^+	

* Thefin itelattices in this set aretop ologically identical.

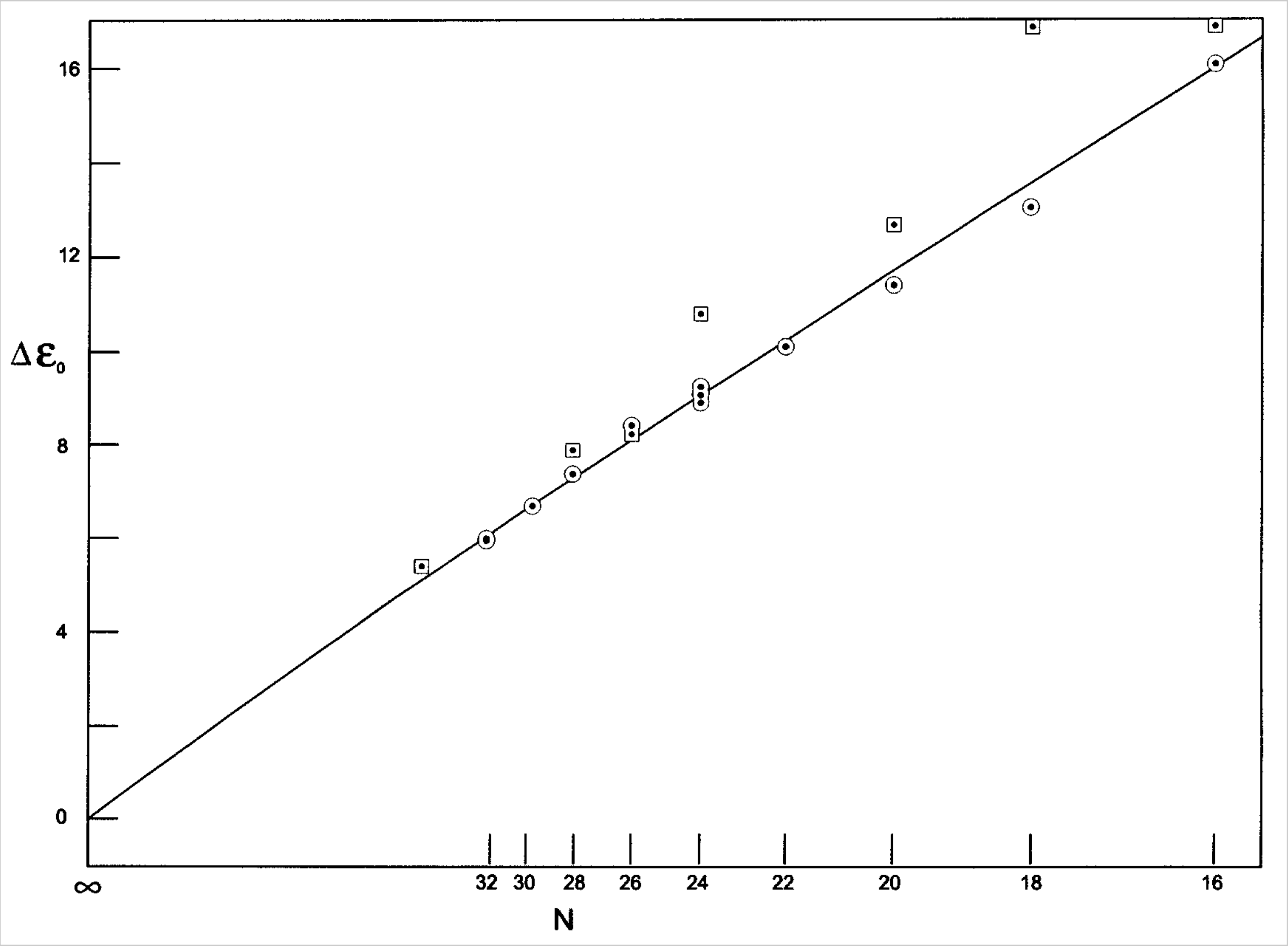
Tile Comparisons

The Spin ½ XY Model



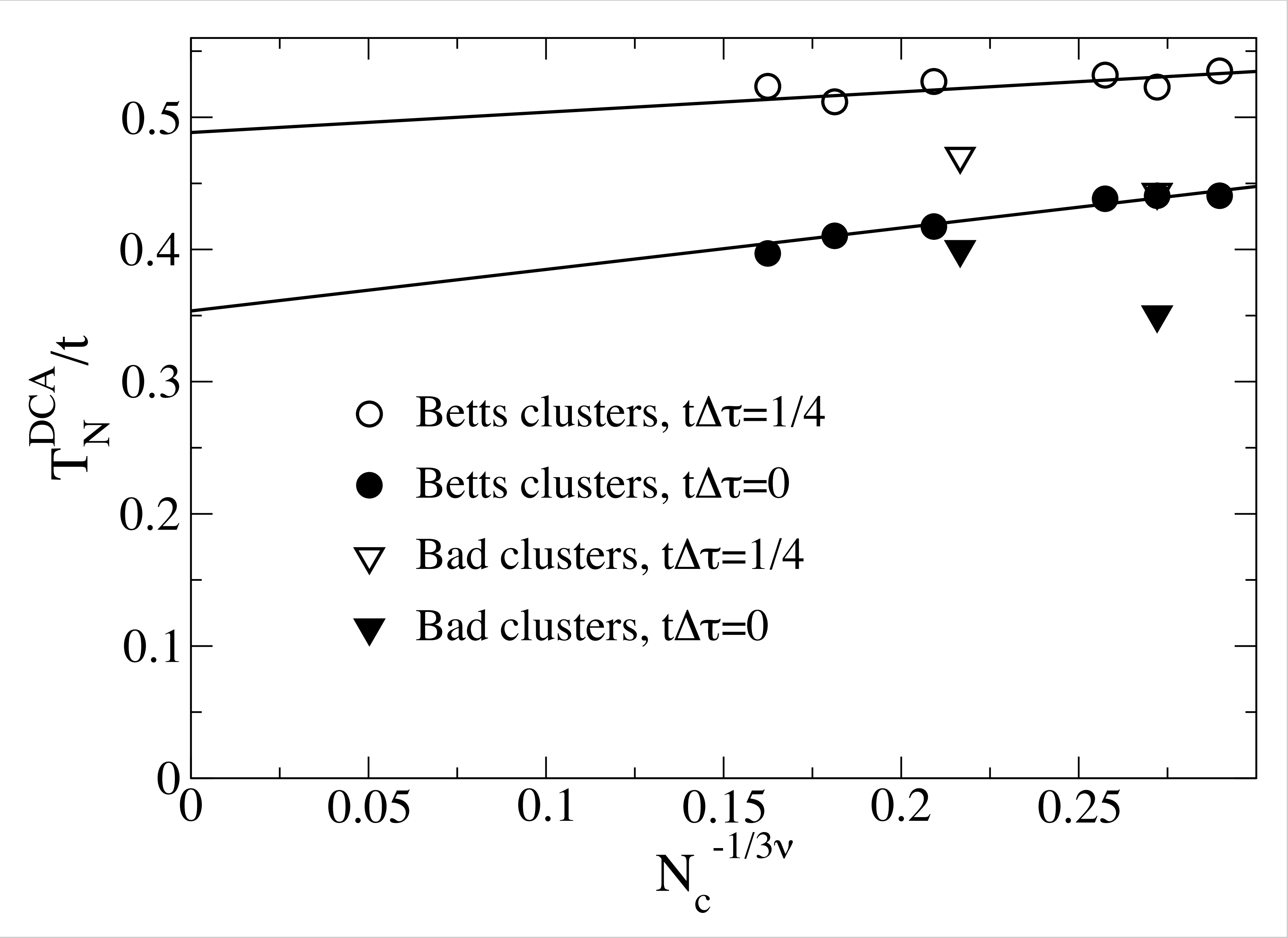
Tile Comparisons

The Spin ½ Heisenberg Model



Tile Comparisons

The Hubbard Model



Comparison with Outside Results

$-\epsilon_0(\infty)$	$m^+(\infty)$	Method	Ref.
0.6657(4)	—	Variational	32
0.6638	—	Variational	33
0.669 68	0.31	Coupled cluster	34
0.669 34(3)	0.3075(25)	Quantum Monte Carlo	35,36
0.669 437(5)	0.3070(3)	Series expansion quantum Monte Carlo	37
0.669 442(26)	0.3077(4)	Green function Monte Carlo	38
0.669 99	0.3069	Third-order spin wave	39
0.669 49	0.30686(10)	Fourth-order spin wave	40
0.668(1)	0.33(3)	t expansion	41
0.6696(3)	0.303(8)	Series expansion	42
0.6693(1)	0.307(1)	Series expansion	43
0.669(1)	0.325	Exact diagonalization on finite lattices	19
0.6513(8)	0.20(1)	"	15
0.669 60(14)	0.302(10)	"	Present estimates

References

1. “Estimation of zero-temperature properties of quantum spin systems on the simple cubic lattice via exact diagonalization on finite lattices”
   D. D. Betts and G. E. Stewart, Can. J. Phys. 75, 47-66 (1997)
2. “Improved finite-lattice estimates of the properties of two quantum spin models on the infinite square lattice”
   D. D. Betts, H. Q. Lin, and J. S. Flynn, Can. J. Phys. 77, 353-369 (1999)
3. “Efficient calculation of the antiferromagnetic phase diagram of the 3D Hubbard model”
   P. R. C. Kent, M. Jarrell, T. A. Maier, and Th. Pruschke, Phy. Rev. B 72 (2005)